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Modeling Partially Coupled Objects with Smooth Particle Hydrodynamics

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C. A. Wingate

MS F663

Los Alamos National Laboratory

Los Alamos, NM 87544

email: caw@lanl.gov

Abstract

A very simple phenomenological model is presented to model objects that are partially coupled (i.e. welded or bonded) where usually the coupled interface is weaker than the bulk material. The model works by letting objects fully interact in compression and having the objects only partially interact in tension. A disconnect factor is provided to adjust the tensile interaction to simulate coupling strengths. Three cases of an example impact calculation are shown - no coupling, full coupling and partial coupling.

Introduction

Smooth Particle Hydrodynamics (SPH) is a gridless, Lagrangian numerical technique to model the fluid equations of motion. SPH has proved to be very useful in certain classes

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of problems, particularly kinetic energy impacts. Our SPH code is called SPHINX and was used for the calculations presented here. If the reader is unfamiliar with SPH and/or the SPHINX code, then the following topics can be accessed.

Brief description of SPH

History of SPH in X-Division

Basic SPH equations

References to SPH review articles

References to some Los Alamos SPH papers

References to SPH strength of materials papers

References to SPH papers on the tensile instability

References to some of J. J. Monaghan's SPH papers

Normally, when two SPH regions get close they become numerically welded or bonded together. This occurs because the particle sums at the interface include particles from both regions. In some cases this is exactly what is desired, however, in other cases this may not be what is needed. For example, two dissimilar pieces of solid material, like aluminum and iron, can push on each other, i.e. interact in compression, but can be pulled apart with no interaction. To simulate this behavior we added a disconnect option in the code so that, in tension, particles in one region are not included in sums for the other region. In compression the sums include particles from both regions. The next two sections discuss the disconnect option and show two calculations, one with disconnects and one without disconnects.

While the disconnect option has proven useful for many problems, there are some situations where the regions should only be partially disconnected rather than fully disconnected. Recently we added a disconnect factor that achieves this behavior. A section below discusses the disconnect factor and shows an example calculation of its use.

Description of Disconnects

Disconnects in SPHINX are a mechanism by which the particles in groups of regions or particles that have been damaged only interact with each other in compression, not in tension. In codes with grids this is similar to a slide line treatment.

Groups are just as the name implies - one or more regions grouped together. It is a very important distinction to know that the disconnect commands only work on groups, they do not work on individual regions. In fact, if a region is not part of a group then it will

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never be disconnected. The default grouping is to have none of the regions put in groups. So it is very important to realize that if you want disconnects, you have to put the regions of interest in groups. The grouping of regions is done with the group command, its format is

```
group group_name region_1 region_2 ... region_n
```

where group_name is an arbitrary group name chosen by the user and region_1, region_2, ... region_n are the names of the regions to be added to the group. The number of regions, n, can be 1 or greater. As an example, take the group_name to be "jewels" and the regions to be "ruby", "diamond" and "emerald". Then the group command to group these three regions together would be

```
group jewels ruby diamond emerald
```

The region names can appear in any order. The same thing can be accomplished with three lines

```
group jewels ruby  
group jewels diamond  
group jewels emerald
```

There are two types of disconnects in SPHINX - disconnecting particles because their groups are different and disconnecting particles because they are damaged. The command for disconnecting particles because their groups are different is

```
disconnect_objects TRUE (or FALSE)
```

Objects is just another word for regions. It would probably be better to call this command "disconnect_groups", but for historical reasons it is "disconnect_objects". The command for disconnecting particles if they are damaged is

```
disconnect_damage TRUE (or FALSE)
```

As an example of the disconnect objects (regions) consider two regions side by side which have also been designated as separate groups. With the disconnects on, the two regions can slide past each other with not interaction and they can separate with no interaction. However, they interact normally when pushing against each other, i.e. in compression.

Calculation With and Without Disconnects

Two calculations were done to illustrate the the use of disconnects, one calculation with

disconnects and one without. A calculation illustrating the use of the `disconnect_factor` is given in the next section.

The initial geometry for all calculations presented here is shown in figure 1 and consists of an aluminum plate impacting a iron cone sitting on top of a iron cylinder. Details of the parameters in the calculation are given in the figure caption.

The issue in this calculation is how to handle the interface between the iron cone and the iron cylinder. The interface can be not disconnected, totally disconnected or partially disconnected. No disconnects at all would simulate a single piece of material with no interface. Full disconnect at the interface would simulate the cone just sitting on top of the cylinder with no bonding or weld between the cone and cylinder. Partial disconnect would simulate a weld or some similar bond between the cone and cylinder where the weld is somewhat weaker than a solid chunk of material would be.

The results from the calculation without any disconnects ("`disconnect_objects FALSE`") is shown in figure 2. The cone and cylinder are acting as one object, there is no separation between the two.

The results from the calculation with full disconnects ("`disconnect_objects TRUE`") is shown in figure 3. The cone has completely separated from the cylinder and there has been very little interaction between the two regions.

Calculation Using the Disconnect Factor

The final calculation presented is the case with partial disconnects at the interface between the cone and the cylinder. This would simulate a weld or some similar bond between the cone and cylinder where the weld is somewhat weaker than a solid chunk of material would be. Of course, it would be much better to actually put weld material between the cone and cylinder and thus model the weld explicitly. This is not done because the detailed weld geometries are not known and the equation of state and strength parameters are not known for welded material. So it makes sense to use a phenomenological model, like the `disconnect_factor`, and try to tie the value of the factor to experiments.

The `disconnect_factor` command specifies a disconnect factor for pairs of regions. The command for the example calculation presented here is

```
disconnect_factor 0.5 cone cylinder
```

where the disconnect factor is 0.5 and the two groups being disconnected are cone and

cylinder. The regions are set up using the `create_region` and `end_region` commands and the groups are set up using the `group` commands. This is shown explicitly below.

```
create_region cone
... Stuff to set up the cone region
end_region
group cone cone

create_region cylinder
... Stuff to set up the cylinder region
end_region
group cylinder cylinder
```

The results from the calculation with partial disconnects ("`disconnect_factor 0.5 cone cylinder`") is shown in figure 4. The cone has partially separated from the cylinder thus simulating the effect of a weak weld.

Disconnect Factor Range

The disconnect factor usually ranges between 0. and 1., however, there is nothing that prevents the user from specifying a factor outside of this range. It is not clear what this means physically and I have not investigated this.

Input File

The input file used for these calculations is available. See input file.

Conclusions

A very simple phenomenological model is presented to model objects that are partially coupled (i.e. welded or bonded) where usually the coupled interface is weaker than the bulk material. The model works by letting objects fully interact in compression and having the objects only partially interact in tension. A disconnect factor is provided to adjust the tensile interaction to simulate coupling strengths. The hope is that the disconnect factor can be set from simple experimental behavior and then used in complex calculations to get better results.

Figure 1

This figure shows the initial configuration for the disconnect example calculation. This is an impact calculation with the projectile being the blue region on the right. This is an Aluminum plate with an initial velocity to the left of 1.0 km/sec. The target consists of two regions, a cone sitting on top of a cylinder. Both the cone and cylinder are made of iron. The equation of state used for all regions is linear us-up (linear_us_up) and the strength model used is elastic perfectly plastic (elastic_perfectly_plastic). The calculation was done in 2D Cartesian coordinates in order to save time and make visualization of the results easier.

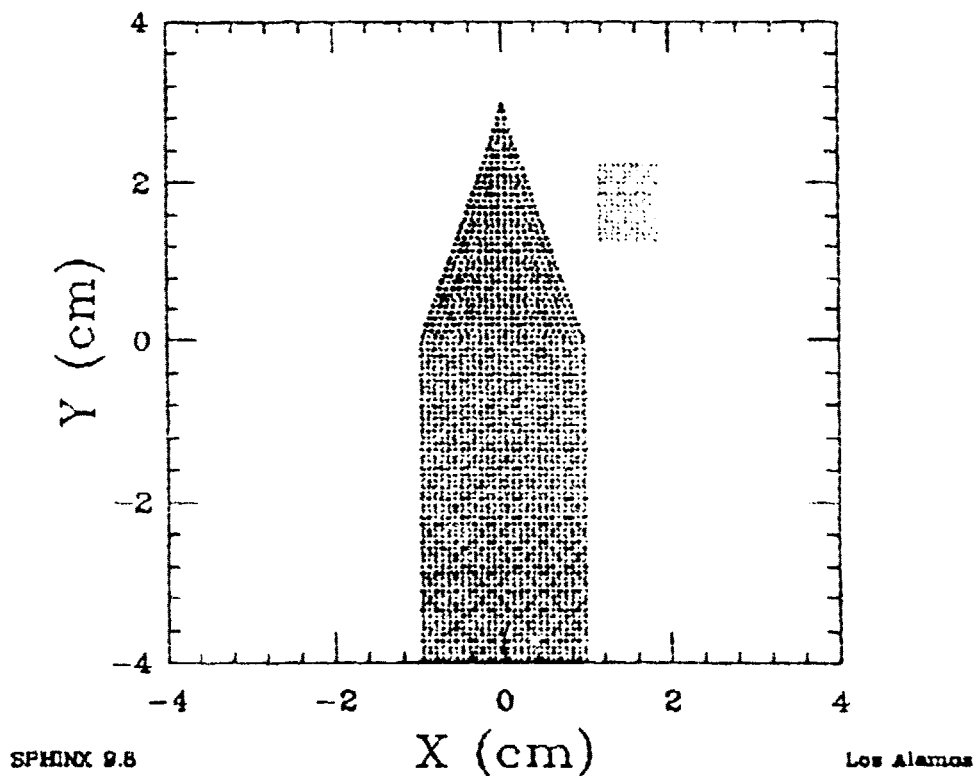


Figure 2

This figure shows the final configuration for the disconnect example calculation. Disconnects are not being used in this calculation. The cone and cylinder targets are behaving as one object, there is no separation between the two.

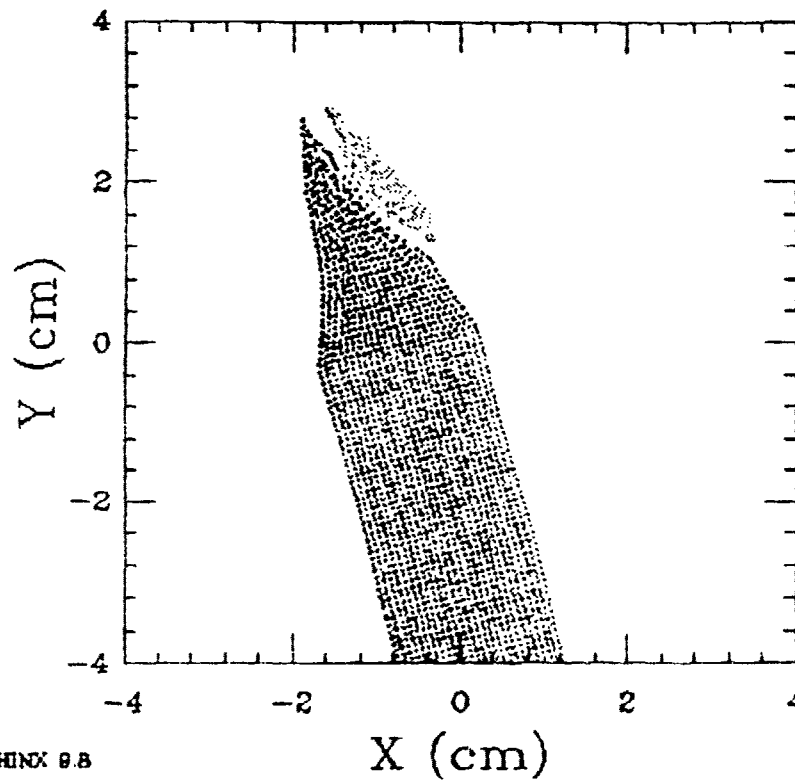


Figure 3

This figure shows the final configuration for the disconnect example calculation where full disconnects are being used between the cone and the cylinder. The cone has completely separated from the cylinder.

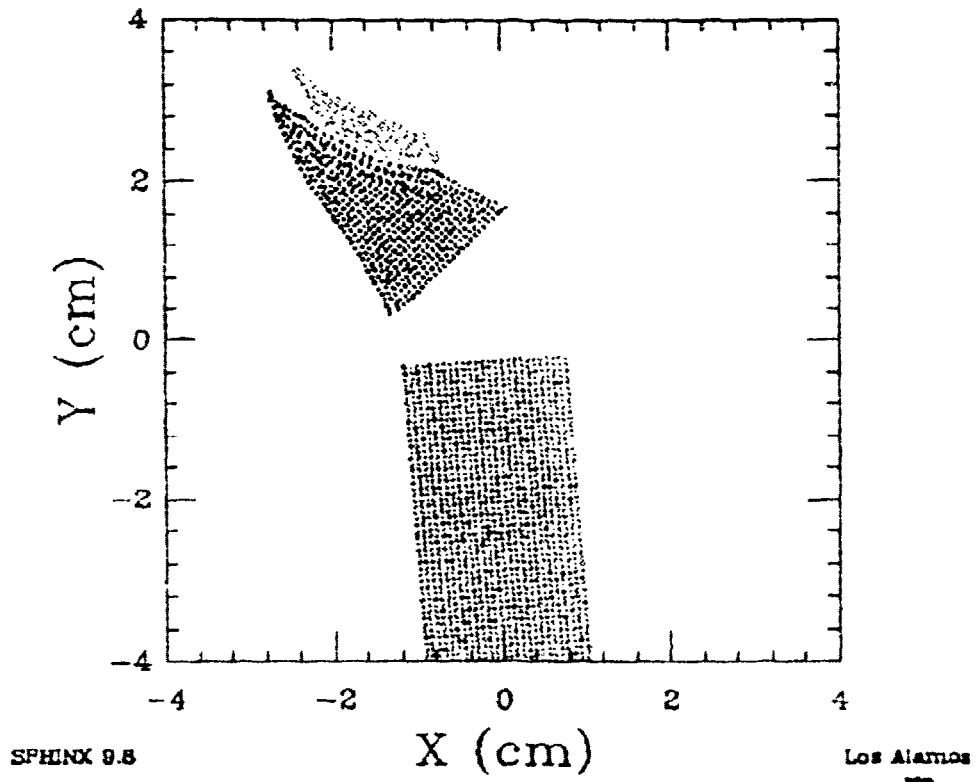
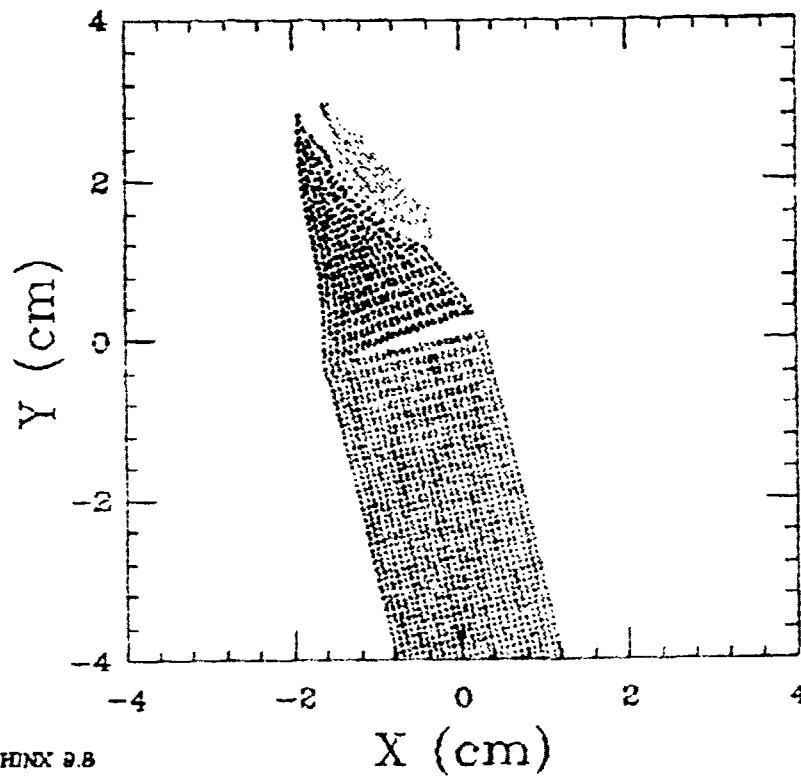


Figure 4

This figure shows the final configuration for the disconnect example calculation where partial disconnects ("disconnect_factor 0.5 cone cylinder") are being used between the cone and the cylinder. The cone has partially separated from the cylinder.



Brief description of SPH

Smooth Particle Hydrodynamics (SPH) is a relatively new technique for hydrodynamic calculations. It is a gridless Lagrangian method using a pseudo-particle interpolation method to compute smooth hydrodynamic variables. Each pseudo-particle has a mass, Lagrangian position and velocity, and internal energy, whereas other quantities are derived by interpolation or from constitutive relations. This technique has now been extended to include strength, high explosives and fragmentation models. The code has been applied to a variety of kinetic energy impact problems including crater formation and target penetration and breakup. Excellent reviews of the SPH method are given in Monaghan (1988 and 1992) and Benz (1989 and 1990) The original work on SPH was done by Lucy (1977). Other reports describing SPH are given by Gingold and Monaghan (1977 and 1982), Monaghan (1982,1985), Stellingwerf (1989a, 1990b) and Stellingwerf and Wingate(1992). Implementation of strength of materials in SPH is described in Libersky and Petschek (1990) and Wingate and Fisher (1993). Some applications of SPH are described in Wingate, et. al. (1992) and Wingate and Stellingwerf (1993).

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SPH History in X-Division

We first got involved with SPH when we imported the SPH code, called SPHC, from Mission Research Corporation (MRC) in Albuquerque in April, 1990. This code was developed by Bob Stellingwerf at MRC. Since then we have exercised the code on a variety of problems and worked with MRC to continue to develop SPHC. Descriptions of the MRC code can be found in Stellingwerf (1989a, 1989b, 1990a, 1990b) and Stellingwerf and Peterkin (1990). In the fall of 1991 the decision was made to write a new SPH code which we called SPHINX (see the SPHINX manual, Wingate, C. A. and R. F. Stellingwerf(1993)). Whereas SPHC was a research code and targeted mostly for PC's and workstations, the new code was written for general use (user friendly input from a data file) and was targeted at first for the Cray (using vectorization to get maximum speed) and later for massively parallel MIMD machines. SPHINX also runs on UNIX workstations but has not yet been put on PC's. SPHC is used as a research tool and currently has more physics and code options built in.

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Equations of Motion

The general fluid dynamics equations are the continuity equation, the momentum equation and the energy equation. The first of these is the continuity equation

$$\frac{d\rho}{dt} + \rho \frac{\partial}{\partial x^\alpha} v^\alpha = 0$$

where ρ is the material density and v is the material velocity. We use Greek superscripts to indicate coordinate directions with implied summation on repeated indices. Roman subscripts will be used to label particles. Summation is not implied on repeated subscripts (the summation sign must appear directly). The second equation is the momentum equation

$$\frac{d}{dt} v^\alpha = \frac{1}{\rho} \frac{\partial}{\partial x^\beta} \sigma^{\alpha\beta}$$

where σ is the stress tensor. This is divided up into an isotropic part which is the pressure and a traceless deviatoric stress tensor and is given by

$$\sigma^{\alpha\beta} = -p \delta^{\alpha\beta} + s^{\alpha\beta}$$

The final equation is the energy equation

$$\frac{de}{dt} = \frac{1}{\rho} \sigma^{\alpha\beta} \dot{\epsilon}_{\alpha\beta}$$

where e is the specific internal energy and $\dot{\epsilon}$ is the strain rate tensor given by

$$\dot{\epsilon}^{\alpha\beta} = \frac{1}{2} \left(\frac{\partial}{\partial x^\beta} v^\alpha + \frac{\partial}{\partial x^\alpha} v^\beta \right)$$

Using the definition of the stress tensor and that the trace of the strain rate tensor is the divergence of the velocity, the energy equation can be rewritten (in perhaps the more familiar form) as

$$\frac{de}{dt} = -\frac{p}{\rho} \frac{\partial}{\partial x^\alpha} v^\alpha + \frac{1}{\rho} s^{\alpha\beta} \dot{\epsilon}_{\alpha\beta}$$

SPH Approximation

The procedure for finding the smooth particle approximation is described in many places, for example see Benz (1989), Benz (1990), Monaghan (1988) or Monaghan (1992). Here we just list the results.

There are two methods for finding the fluid mass density . The first, is done by solving the continuity equation directly

$$\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_j} (\mathbf{v}_i^\alpha - \mathbf{v}_j^\alpha) \frac{\partial W}{\partial x_i^\alpha}$$

and the other, called the sum method, is to use the sum of the masses times the kernel

$$\rho_i = \sum_j m_j W_{ij}$$

Only one of these methods can be used to calculate the density for the fluid equations of motion. The sum method is simpler but has a problem at interfaces where it will smooth out the density and at boundaries where the density will drop. This in turn leads to spurious pressures from the eos package which will usually cause serious problems as the calculation proceeds. Solving the continuity equation does not have this problem and because of this it is the recommended method for most problems.

The momentum equation in the SPH approximation becomes

$$\frac{d}{dt} v_i^\alpha = \sum_j m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} \right) \frac{\partial W}{\partial x_i^\beta}$$

where m subscript j is the mass of particle j and W is the smoothing kernel. The kernel should be written as to indicate its dependence on both particles i and j and its dependence on the smoothing length h . For simplicity, however, we will write it simply as W . The kernel usually used is the cubic B-spline. The energy equation in the SPH approximation is

$$\frac{d\epsilon_i}{dt} = \left(\frac{P_i}{\rho_i^2} \sum_j m_j (\mathbf{v}_i^\alpha - \mathbf{v}_j^\alpha) \frac{\partial W}{\partial x_i^\alpha} \right) + \frac{\sigma_i^{\alpha\beta}}{\rho_i} \epsilon_i^{\alpha\beta}$$

To obtain the particle approximation for the strain rate tensor we follow Libersky and Petschek (1990) to get

$$\epsilon_i^{\alpha\beta} = \frac{1}{2} \sum_j^m \frac{m_j}{p_j} \left((v_j^\alpha - v_i^\alpha) \frac{\partial W}{\partial x_i^\beta} + (v_j^\beta - v_i^\beta) \frac{\partial W}{\partial x_i^\alpha} \right)$$

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Input File for the Example Calculation

This is the input file that was used for the example impact calculations presented in this note. Version 9.8 of SPHINX was used to run the calculations.

```
/* Test the disconnect factor.
*/

{
  /* Titles */
  problem_title disconnect;
  run_title 04;

  /* Debug */
  //debug_global TRUE;
  //debug_flow TRUE;

  /* Density */
  density_method continuity; //sum_mW
  /* dump_variables sumW */

  /* Disconnects */
  disconnect_objects TRUE;
  disconnect_factor 0.5 cone cylinder

  /* Dumps */
  restart_dump_deltat 0.5e-06;
  restart_dump_format cdat;
  restart_dump_type binary;

  plot_dump_deltat 20.0e-06;
  plot_dump_format cdat;
  plot_dump_type binary;
  dump_variables csq nn_per_point;
  dump_reflected_particles TRUE;

  history_dump_deltat .1e-12;
  history_dump_type ascii;
}
```

```

/* Control */
max_steps 0;
max_time 200.0e-06;
step_print_delta 1;

/* Geometry */
dimension 2;
cylindrical FALSE;
use_grid_gen TRUE;

/* Graphics */
graphics FALSE;
particle_plot_cycle_frequency 1;
plot_xminmax 1 -3. 3.;
plot_yminmax 1 -3. 3.;
color_plot_variable pressure;
plot_reflected_particles TRUE;

/* H */
h_vary TRUE;
h_inp 1.5;
heff_fsteps 1.2;
average_nn 40;

/* History */
Lagrangian_probe 0. 0. 0.;
probe_dump_variables yield shear_modulus;
probe_dump_variables sigma_th;
probe_dump_variables sqrt_J2;

/* Particles */
nparticles 3000;
nparticle_mult 0.9;

/* Strength */
strength TRUE;
dump_variables J1 sqrt_J2 maxps eps_equiv epsdot_equiv;
dump_variables yield shear_modulus stressd;

/*****
***** Region Setup *****/
/*****/
/* Set up the projectile*/
create_region projectile {
  density 2.7;
  velocity 0. 1.00e+05 0.;
  type cylinder radius 0.1 length 0.75;
  strength_material al_6061_t6;
  strength_model elastic_perfectly_plastic;
  eos_material al;

```

```

    eos_model          linear_us_up;
)
end_region;
rotate_region projectile 0. 0. 90.;
translate_region projectile 0. 0.25 0.;
translate_region projectile 0. 1.5 0.;
translate_region projectile 1.5 0. 0.;
//group projectile projectile;

/* Set up target - this is a cone on top of a cylinder */
create_region cylinder {
    density          7.85;
    type             cylinder radius 1.0 length 5.0;
    strength_material fe_armco;
    strength_model   elastic_perfectly_plastic;
    eos_material     fe;
    eos_model        linear_us_up;
}
end_region;
translate_region cylinder 0. -2.5 0.;
group cylinder cylinder;

create_region cone (
    density          7.85;
    type             cone radius 1.0 length 3.0;
    strength_material fe_armco;
    strength_model   elastic_perfectly_plastic;
    eos_material     fe;
    eos_model        linear_us_up;
)
end_region;
translate_region cone 0. 1.5 0.;
group cone cone;

/*****
***** Strength Parameters *****/
strength_parameters elastic_perfectly_plastic al_6061_t6 {
    shear_modulus  2.65e11;
    yield          2.75e09;
}
end_strength;

strength_parameters elastic_perfectly_plastic fe_armco {
    shear_modulus  8.e11;
    yield          6.e09;
}
end_strength;

```

```

/*****
***** Equation of State Parameters *****/
eos_parameters linear_us_up al (
  c0          5.376e5;
  s           1.55;
  gamma0      2.10;
  gamma1      0.;
  solid_density 2.7;
  pmin        -6.e09;
)
end_eos;

eos_parameters linear_us_up fe (
  c0          5.958e5;
  s           1.5175;
  gamma0      2.035;
  gamma1      0.;
  solid_density 7.85;
  pmin        -10.0e9;
)
end_eos;

```

SPH Review References

These are references to SPH reviews.

Benz, W. (1989). Smooth Particle Hydrodynamics: A Review. Harvard-Smithsonian Center for Astrophysics Preprint No. 2884

Benz, W. (1990). Smooth Particle Hydrodynamics: A Review. J. R. Buchler (ed.), The Numerical Modeling of Nonlinear Stellar Pulsations, 269-288.

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SPH Strength References

These are references relating to strength of materials in SPH.

Libersky L. D., and A. G. Petschek, (1990). Smooth Particle Hydrodynamics with Strength of Materials, *Advances in the Free-Lagrange Method*, (Trease, Fritts, and Crowley, eds.), Springer Verlag, 248

Wingate, C. A. and H. N. Fisher (1993). *Strength Modeling in SPHC*. Los Alamos National Laboratory report, LA-UR-93-3942.

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SPH Instability References

These are references relating to the tensile instability in SPH.

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Dyka, C. T. and R. P. Ingel (1994). Addressing Tension Instability in SPH Methods. Naval Research Laboratory Report, NRL/MR/6384--94-7641.

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Monaghan, J. J. and J. A. Morris (1993). SPH Masters Negative Stress. In the proceedings of the "Workshop on Advances in Smooth Particle Hydrodynamics", Los Alamos National Laboratory Report, LA-UR-93-4375.

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